

IES 302: Quiz 2 Solution

1) $Z \sim \mathcal{N}(0,1)$.

a) $P[Z > 1.08] = 1 - P[Z \leq 1.08] = 1 - \Phi(1.08) = \boxed{0.140071}$

b) $P[Z > -1.08] = P[Z < 1.08] = \Phi(1.08) = \boxed{0.859929}$

c)

$$\begin{aligned} P[-1.08 < Z < 1.08] &= \Phi(1.08) - \Phi(-1.08) = \Phi(1.08) - (1 - \Phi(1.08)) \\ &= 2\Phi(1.08) - 1 = \boxed{0.719858}. \end{aligned}$$

2) $X \sim \mathcal{N}(9,9)$.

a) $P[X > 10] = P\left[\frac{X-9}{3} > \frac{10-9}{3}\right]$. Note that for $X \sim \mathcal{N}(\mu, \sigma^2)$, we have

$$Z = \frac{X - \mu}{\sigma} \sim \mathcal{N}(0,1). \text{ So,}$$

$$P[X > 10] = P\left[Z > \frac{1}{3}\right] = 1 - P\left[Z \leq \frac{1}{3}\right] = 1 - \Phi\left(\frac{1}{3}\right) = \boxed{0.370700}.$$

b) $P[X > -5] = P\left[Z > \frac{-5-9}{3}\right] = P\left[Z < \frac{14}{3}\right] = \Phi\left(\frac{14}{3}\right) = \Phi(4.67)$. Note that $z = 4.67$ is

beyond what the Φ table provided. However, we can see that at 3.99, the value of Φ is almost 1. Hence, $\Phi(4.67) \approx \boxed{1}$.

c)

$$\begin{aligned} P[-5 < X < 10] &= P\left[\frac{-5-9}{3} < Z < \frac{10-9}{3}\right] = \Phi\left(\frac{1}{3}\right) - \Phi\left(-\frac{14}{3}\right) \\ &= \Phi\left(\frac{1}{3}\right) - \left(1 - \Phi\left(\frac{14}{3}\right)\right) = \Phi\left(\frac{1}{3}\right) + \Phi\left(\frac{14}{3}\right) - 1 \\ &\approx \Phi\left(\frac{1}{3}\right) = \boxed{0.629300}. \end{aligned}$$